



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Lecture 8: Basic Optics for Optical Sensing-VI

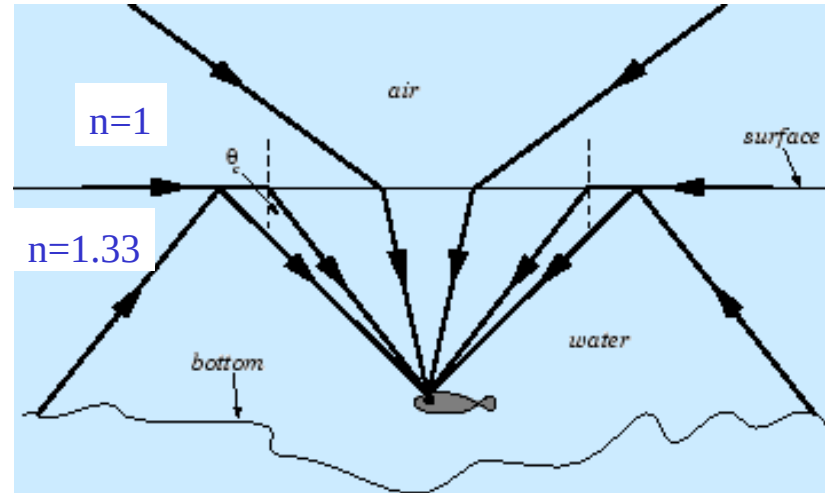
Evanescent sensors, Absorption and dispersion: Drude model for conductors

Sachin Kumar Srivastava

Department of Physics

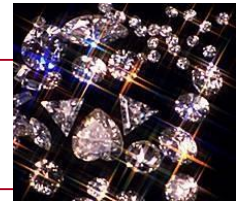


TIR, Fish Vision and Diamonds Brilliance



The fish sees the sky if it looks upwards but sees the bottom of the pond if it looks at an angle larger than 48.8° .

Question: explain the brilliance of diamond in air knowing that for diamond $n=2.42$ and explain why it is less shiny when it is in water?



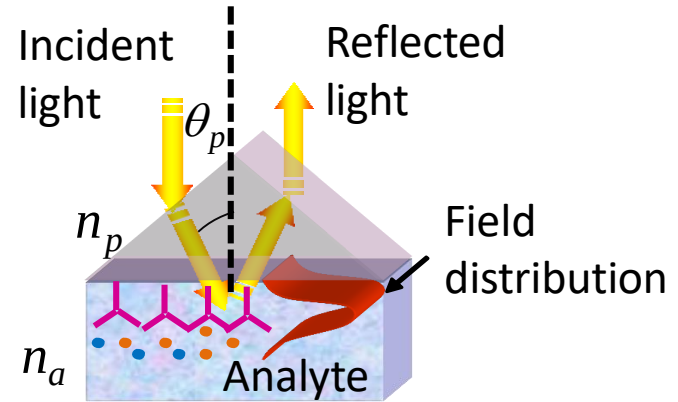
Can we do Sensing using TIR?

$$\sin \theta_c = \frac{n_a}{n_p}$$

Use: $\frac{d \sin x}{dx} = \cos x$

$$\frac{d \sin \theta_c}{dn_a} = \cos \theta_c \frac{d \theta_c}{dn_a} = \frac{1}{n_p} \Rightarrow$$

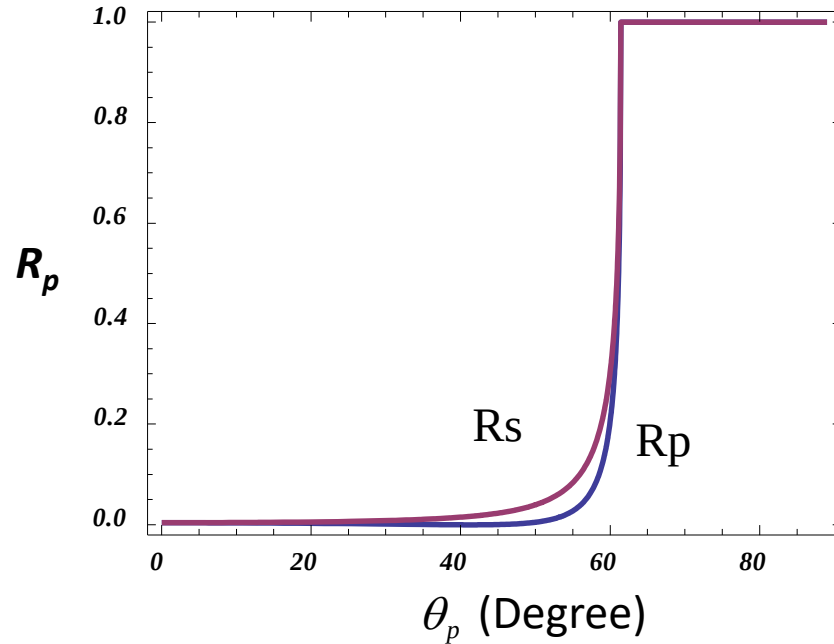
$$S = \frac{d \theta_c}{dn_a} = \frac{1}{n_p \cos \theta_c} = \frac{1}{n_p \sqrt{1 - \sin^2 \theta_c}} = \frac{1}{n_p \sqrt{1 - n_a^2 / n_p^2}} = \frac{1}{\sqrt{n_p^2 - n_a^2}}$$



Example: BK7 prism and
water analyte:

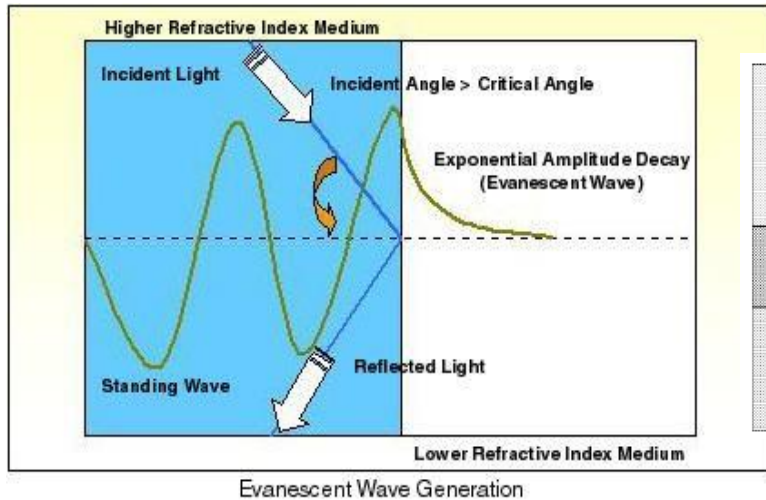
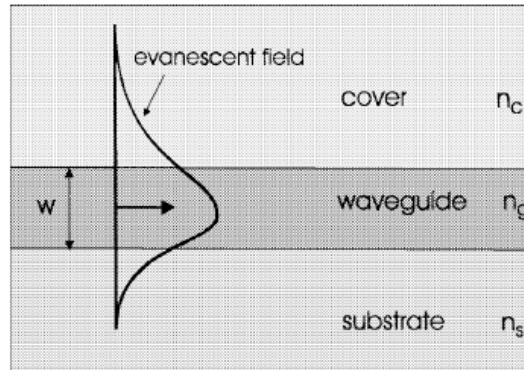
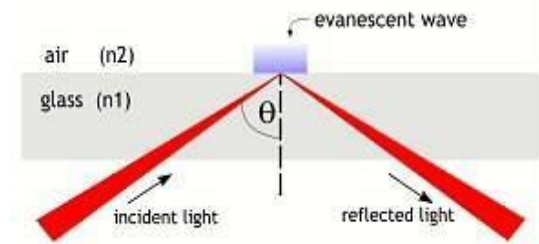
$$S = \frac{1}{\sqrt{n_p^2 - n_a^2}} = \frac{1}{\sqrt{1.515^2 - 1.33^2}}$$
$$\approx 1.38 \text{ rad} / \text{RIU} = 78.97 \text{ deg} / \text{RIU}$$

High sensitivity and sharp
response Curve!



Evanescent Wave Sensors

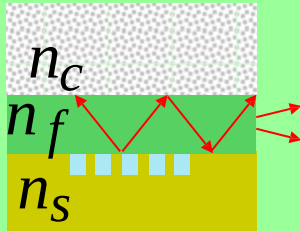
The sensing is performed by the evanescent tail of the modal field in the cover medium. This sensing operation consists of measuring the change of the effective index of a propagating mode when a change of the refractive index takes place in the waveguide cover. Or due to absorption!



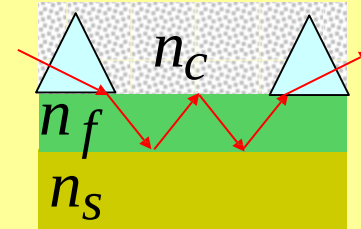
Evanescent wave sensor configurations

Wave guide geometries

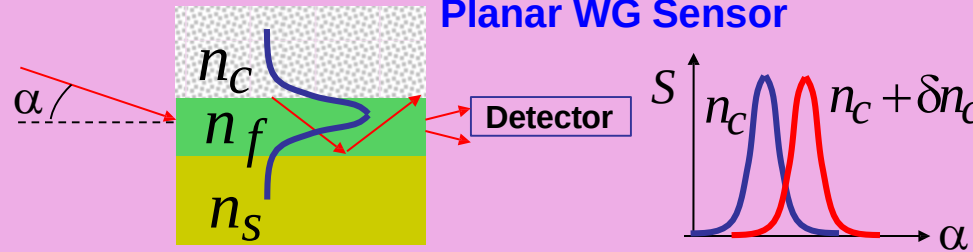
Grating coupled WG Sensor



Prism coupled WG Sensor

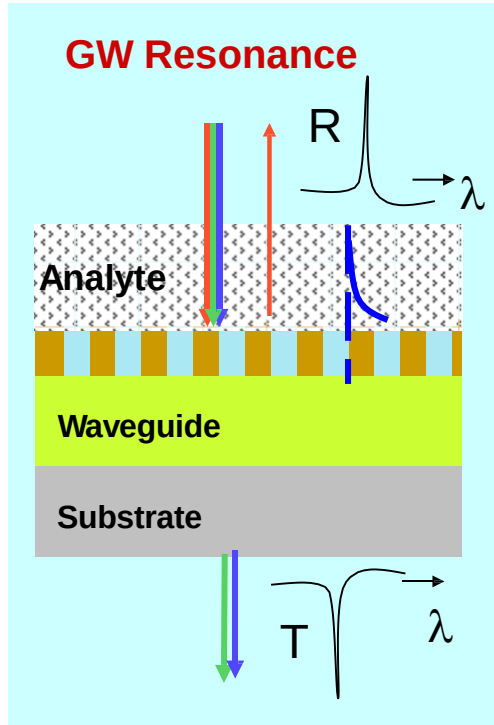
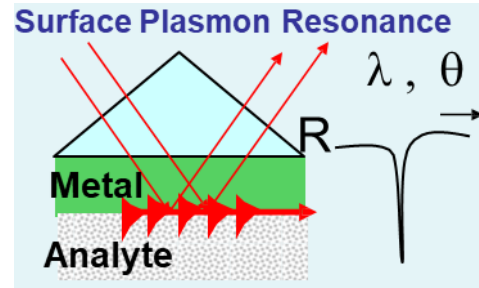
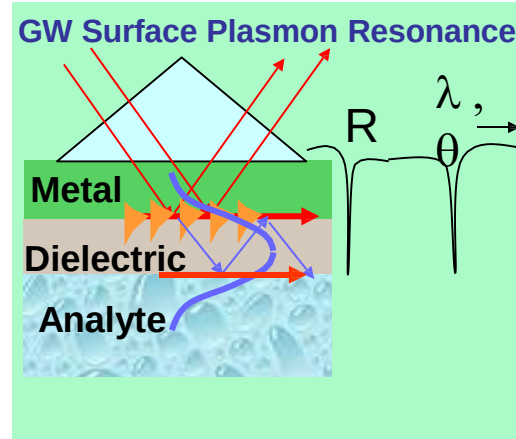


Planar WG Sensor

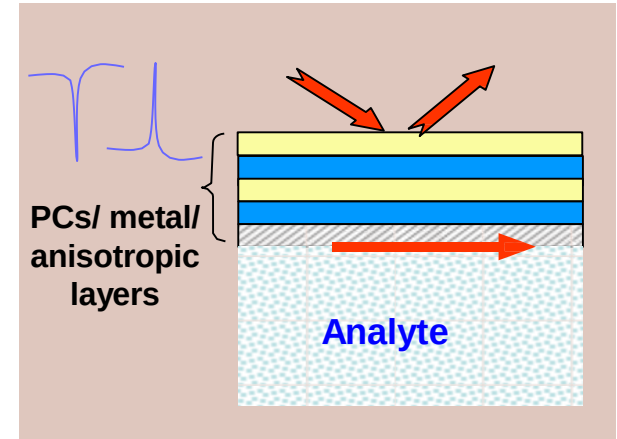


Evanescent wave sensor configurations

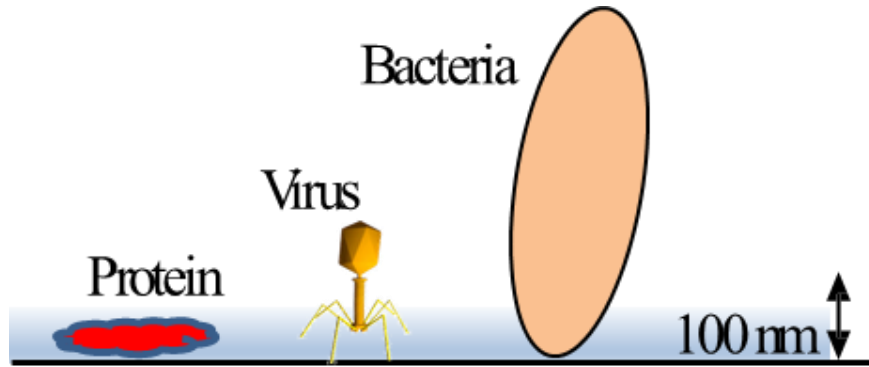
Plasmonic sensors



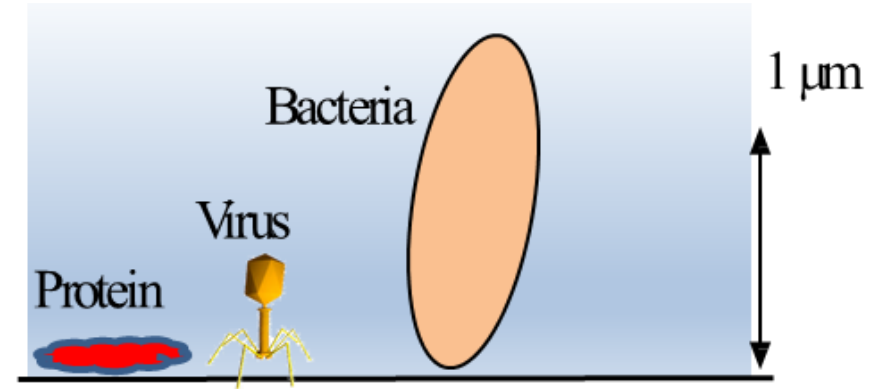
SEW from PCs & anisotropic films



The importance of the Penetration Depth



(b)



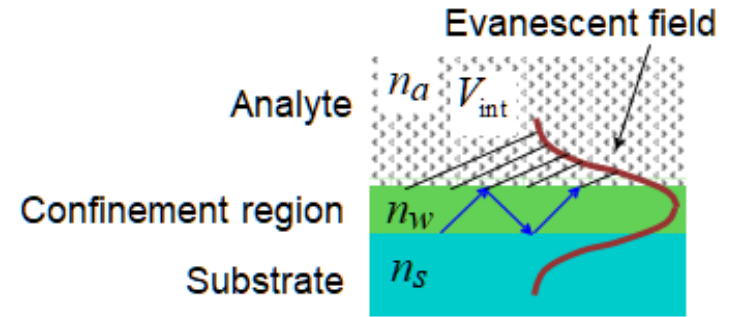
(a)

Analytical Approach towards Evanescent Field Sensing

$$\varepsilon = \begin{cases} n_w^2 & r \in V_w \\ n_{a,s}^2 & r \notin V_w \end{cases}$$

Assuming a particle is added to the analyte, it creates a variation in the dielectric function:

$$\delta\varepsilon = \begin{cases} n_p^2 - n_a^2 & r \in V_{\text{int}} \\ 0 & \text{otherwise} \end{cases}$$



The wave vector will change by: $\delta k = k_f - k_i$ and the field E_i to E_f

Evanescent Field Sensing

1st Order Perturbation approach

Wave equation before the perturbation:

$$\nabla_x \nabla_x E_i = k_i^2 \epsilon E_i$$

The wave equation after the particle is added:

$$\nabla_x \nabla_x E_f = k_f^2 (\epsilon + \delta\epsilon) E_f$$

Multiplying by E_i^* and integrating over the entire volume and subtracting yields:

$$(k_i^2 - k_f^2) \int_V E_f \cdot \epsilon E_i^* dr = k_f^2 \int_{V_{\text{int}}} E_i^* \cdot \delta\epsilon E_f dr$$



➡ Up to the first order in: δk

$$\delta k \approx \frac{k_i}{2} \frac{\int_V \delta \epsilon E_i^* \cdot E_f dr}{\int_V \epsilon E_i^* \cdot E_i dr}$$

The shift in the wave vector is equal to the overlap integral normalized by the mode energy integral. Sensing in the evanescence region!

Main Parameters Affecting the Sensitivity

$$\frac{\delta k}{k_i} = \frac{\delta n_{eff}}{n} = \frac{\delta \lambda}{\lambda} \approx - \frac{1}{2} \frac{V_{int} \int \delta \epsilon E_i^* \cdot E_f dr}{\int_V \epsilon E_i^* \cdot E_i dr}$$

Field Strength

Interaction volume

Sensors geometry, nano-structures, localization, band gaps, material parameters

Evanescence region, lateral propagation length, porosity

Absorption

When the refractive index is complex: $n = n_r \pm i\kappa$

κ -The extinction coefficient !. The sign depends on the choice: $e^{\pm i\kappa z}$

The relation to real and imaginary parts of the dielectric function:

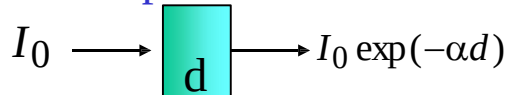
$$\varepsilon_r = n_r^2 - \kappa^2 \text{ and } \varepsilon_i = 2n_r\kappa$$

A plane wave will be decaying as: $E(z) = \exp(ik_0 n_r z) \exp(-k_0 \kappa z)$

The intensity is decaying as: $|E|^2 = \exp(-2k_0 \kappa z)$

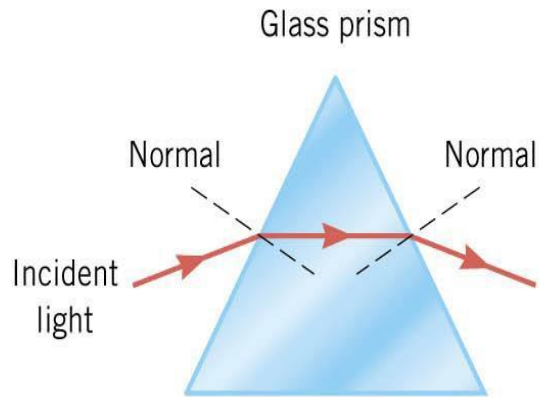
The absorption coefficient is defined as:

$$\alpha = 2k_0 \kappa = \frac{4\pi\kappa}{\lambda_0}$$

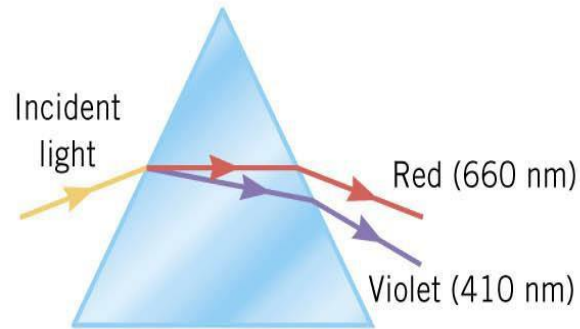


If we ignore reflections at the boundaries!

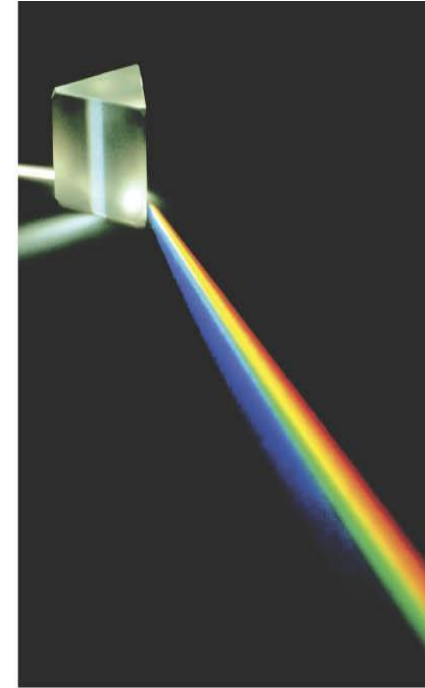
Dispersion - Prism



(a)

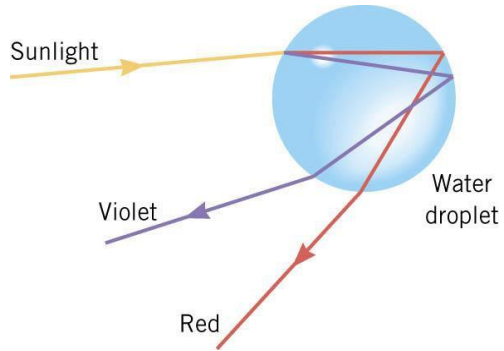
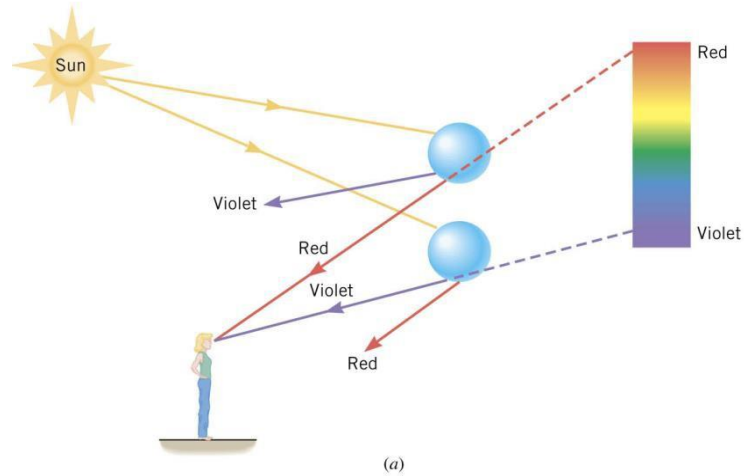


(b)



(c)

Dispersion - Rainbow



(b)

Dispersion – Lorentz Model

Oscillating bound electron model (Lorentz model):

$$m_e(r(t) + \gamma r(t) + \omega_0^2 r(t)) = -q_e E_{eff}(t)$$

inertia

damping

restoring
force

applied
force

Now use:

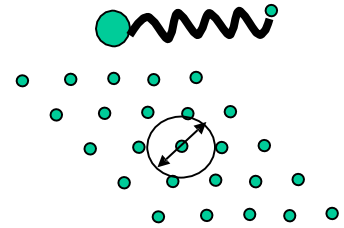
$$E_{eff}(r, t) \approx E(r, t) + \frac{4\pi P(r, t)}{3}$$

and

$$P(r, t) = N\mu_a = -Nq_e r(t)$$

Mossotti (1850) equation (CGS)

Depolarization field



Dispersion – cont'

Then:

$$\ddot{P}(t) + \gamma\dot{P}(t) + (\omega_0^2 - \frac{4\pi Nq_e^2}{3m_e})P = \frac{Nq_e^2}{m_e} E(t)$$

Use: $P(t) \propto \exp(i\omega t)$ and: $D = \epsilon E = E + 4\pi P$ (CGS)

To get:

$$\epsilon(\omega) \approx 1 + \frac{b^2}{\omega_{0s}^2 - \omega^2 - i\gamma\omega}$$

Where:

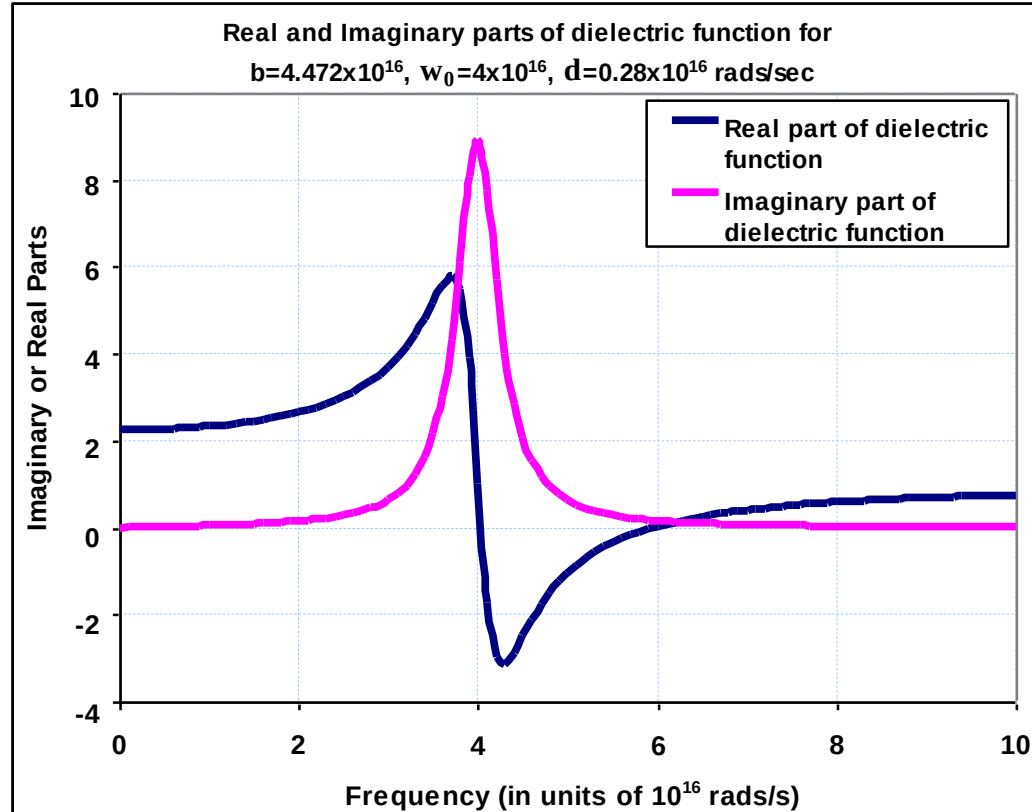
$$\omega_{0s} = \sqrt{\omega_0^2 - \omega_c^2}$$



Resonance frequency is shifted by frequency of oscillating dipoles

$$\omega_c^2 = b^2/3 = 4\pi Nq_e^2/3m_e$$

Dispersion – cont'



RI Dispersion in Glasses

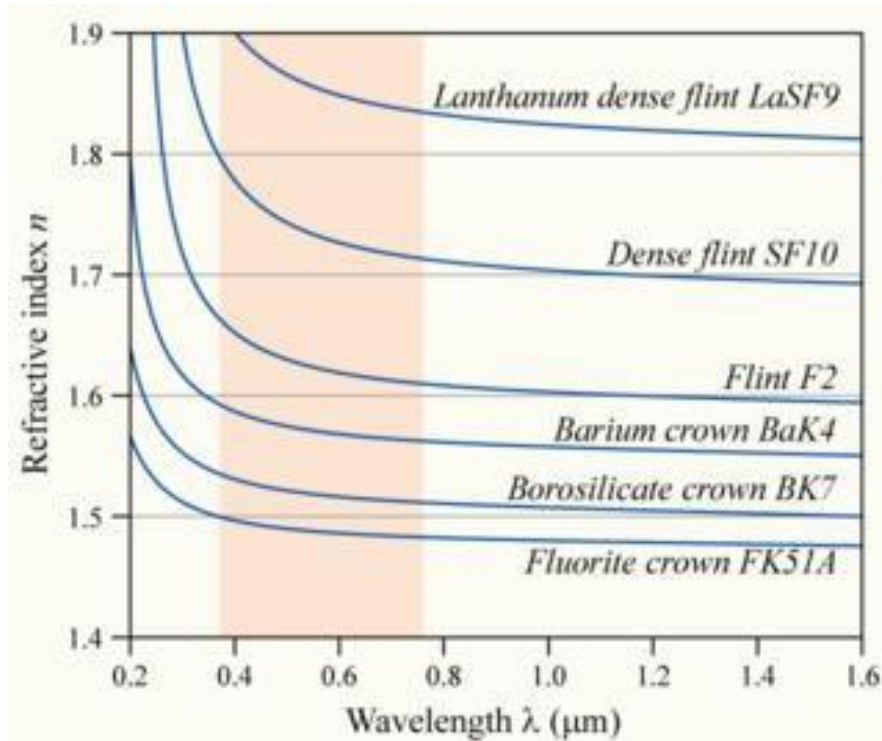
Glasses are usually transparent in order to be used for imaging, hence their refractive indices are real and follow one of the following formulae:

Cauchy:
$$n(\lambda) = n_{\infty} + \frac{a}{\lambda^2} + \frac{b}{\lambda^4} + \dots$$

Sellmeier:
$$n(\lambda) = \sqrt{1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} + \dots}$$

$$n(\lambda) = \sqrt{A + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} + \dots}$$

RI Dispersion in Glasses II



For BK7:

Coefficient	Value
B_1	1.03961212
B_2	0.231792344
B_3	1.01046945
C_1	$6.00069867 \times 10^{-3} \mu\text{m}^2$
C_2	$2.00179144 \times 10^{-2} \mu\text{m}^2$
C_3	$1.03560653 \times 10^2 \mu\text{m}^2$

See one of the useful websites:

<http://refractiveindex.info/>

Dispersion of Free Carriers

In space with free charge carriers such as free electrons in metals or ions in the ionosphere no restoring force and no damping hence the equation we derived for the oscillating dipole becomes:

$$\ddot{P}(t) = \frac{Nq_e^2}{m_e} E(t) \quad \Rightarrow \quad P(t) = \frac{-Nq_e^2}{m_e \omega^2} E(t)$$

Use:

$$D = \epsilon E = E + 4\pi P \quad \Rightarrow \quad \epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

Where the plasma frequency:

$$\omega_p^2 = 4\pi N q_e^2 / m_e = b^2 \quad (\text{CGS})$$

$$\omega_p^2 = N q_e^2 / \epsilon_0 m_e \quad (\text{SI})$$

Dispersion of free Carriers– cont'

From the relation for bound electrons:

$$\varepsilon(\omega) \approx 1 + \frac{b^2}{\omega_{0s}^2 - \omega^2 - i\gamma\omega}$$

Substitute $\omega_{0s} = 0$ for free carriers to get:

$$\varepsilon(\omega) \approx 1 - \frac{b^2}{\omega(\omega + i\gamma)} = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)} \xrightarrow{\gamma=0} 1 - \frac{\omega_p^2}{\omega^2}$$

↑
Drude Term

Which is the same function obtained without damping

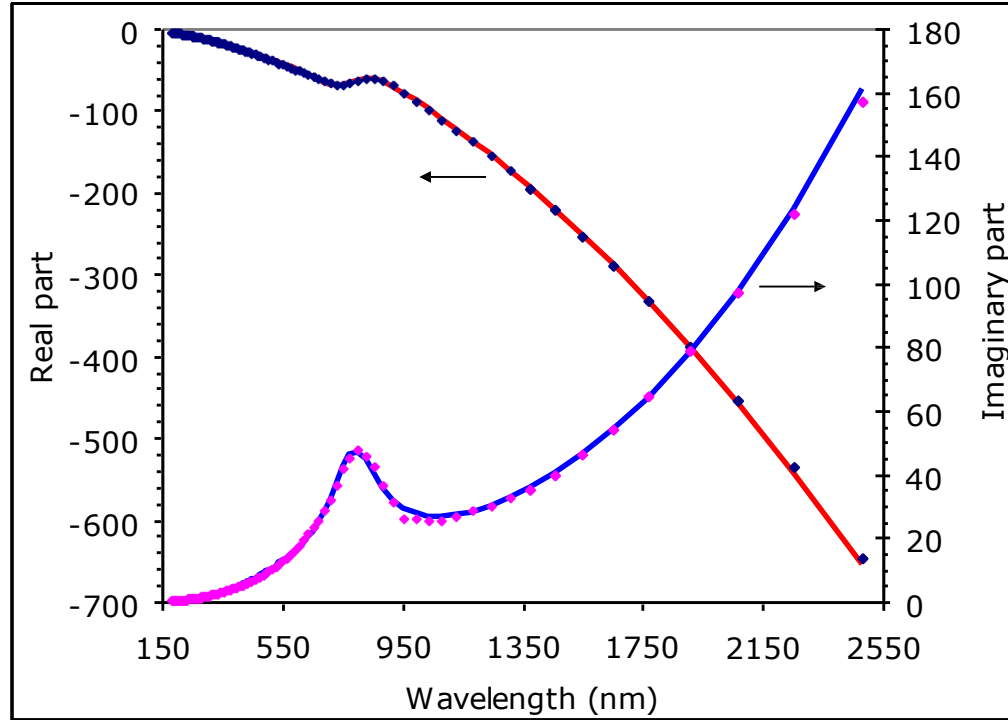
Dispersion of Metals

Better described by a combination of Drude term and two oscillators:

$$\varepsilon(\omega) = \varepsilon_{\infty} - \frac{\omega_p^2}{\omega^2 + i\Gamma_p\omega} + \sum_j \frac{f_j\omega_j^2}{\omega_j^2 - \omega^2 + i\Gamma_j\omega}$$

Metal	ε_{∞}	$\omega_p(\text{eV})$	$\Gamma_p(\text{eV})$	f_1	$\omega_1(\text{eV})$	$\Gamma_1(\text{eV})$	f_2	$\omega_2(\text{eV})$	$\Gamma_2(\text{eV})$
Ag	2.302	8.807	0.082	0.439	4.602	0.761	1.098	6.008	2.393
Au	3.952	8.542	0.070	-85.116	2.6	3.712	90.046	2.789	4.27
Al	0.952	13.335	0.117	8.119	1.579	0.407	9.908	2.006	1.942

Fit for Aluminum



Measurements are based on spectroscopic ellipsometry of Sopra (<http://www.sopra-sa.com/>)

Summary

1. Analytical approach for evanescent wave-based sensing was introduced and the factors affecting performance were discussed.
2. Dispersion characteristics of metals were discussed and Drude model was achieved.

Thank You